Complex numbers provide mathematics to handle two related quantities that have some degree of independence from each other.

Complex numbers have two parts: a real part and an imaginary part. This is represented mathematically by $a + jb$, where a and b are real numbers and j is the square root of -1. If a is zero, then the number is called an imaginary number. If b is zero, the number is called a real number.

In complex numbers, j is called an operator. A value of j will rotate a vector by 90 degrees counterclockwise. Each additional power of j will rotate a vector by an additional 90 degrees counterclockwise. The quantity $j1$ is rotated by 90 degrees, while the quantity $j^2 1$ is rotated 180 degrees. This pattern continues through $j^4 1$, which returns to the original vector angle, since j to the fourth power is 1 (the square root of -1 squared is -1, and -1 squared is 1).

Mathematicians write complex numbers using i as the square root of -1, where engineers write complex numbers using j as the square root of -1, since engineers use i to represent current flow.

Graphical Representations of Complex Numbers

A two-axis coordinate system representing complex numbers has the x axis representing the real portion of complex numbers and the y axis representing the imaginary portion of complex numbers. This system, called the complex plane, labels the x axis with R and the y axis with j.

Notation of Complex Numbers

There are two typical ways of notating complex numbers. Rectangular notation represents the x axis as the real portion (the a in $a + jb$) and the y axis as the imaginary portion (the b in $a + jb$). Polar notation represents complex numbers as a magnitude and an angle (direction), in essence as a vector, for example, 10 at angle 15.

Conversion between Notations

You can convert rectangular notation to polar notation by using the Pythagorean theorem to calculate the magnitude from the real and imaginary coefficients and the angle from the arc tangent of (b / a). You can convert polar notation to rectangular notation by using trigonometry to calculate the real coefficient a by the magnitude times the cosine of the angle and the real coefficient b by the magnitude times the sine of the angle.

Mathematical Operations on Complex Numbers

Add complex numbers using rectangular notation by adding the a coefficients together separately from adding the *b* coefficients together.

Subtract complex numbers using rectangular notation by subtracting the corresponding *a* coefficients separately from subtracting the corresponding *b* coefficients.

Multiply complex numbers in rectangular notation using binomial multiplication and subsequent simplification. Multiply complex numbers in polar notation by multiplying the magnitudes and adding the angles.

Divide complex numbers in rectangular notation by:

1. calculating the complex conjugate of the divisor by changing the sign of the imaginary coefficient
2. multiplying both the dividend and the divisor of the number by the complex conjugate of the divisor

Divide complex numbers in polar notation by dividing the magnitudes and subtracting the angle of the divisor from the angle of the dividend.