

## RC Time Constant

The RC time constant is a useful metric in electronics. It quantifies the behavior of a circuit consisting of a resistor and a capacitor in series (called a series RC circuit) when an external voltage is applied to charge the capacitor, and when the capacitor is allowed to discharge through the resistor. The RC time constant is used in video and audio electronics.

Before explaining the RC time constant, let's learn about the mathematics that models how it works.

### Exponential Decay

When the amount of some material increases or decreases at a rate that is proportional to the amount of material at the time, the rate of change is called exponential. This is different than a linear rate of change, where the rate of change is constant and independent of the amount of the material undergoing change.

Exponential decay is a good thing for engineers and scientists to understand. You'll see it in numerous examples in physics, such as:

- the decay of radioactive materials
- heating and cooling of substances according to Newton's law
- decreases in atmospheric pressure that occur with increasing altitude
- sounds and other vibrations that gradually decrease in amplitude due to the presence of a countering force

The behavior of an RC circuit also exemplifies exponential decay, which is why we are talking about it.

Exponential decay is based on Euler's number.

### Euler's Number aka Napier's Constant aka $e$

The number  $e$  is one of a few special numbers in mathematics. It cannot be expressed as a ratio of two numbers (it is irrational), and it is a nonrepeating and nonterminating number that has no maximum precision (i.e., if you try to write it, you'll never stop). It is approximately equal to 2.7182818.

You already are familiar with other numbers that have these qualities:

- pi (the ratio of a circle's circumference to its diameter)
- the sine, cosine, and tangent of any nonzero rational number

### Natural Logarithms

Logarithms having  $e$  as a base are called natural logarithms. They are notated

## RC Time Constant

by  $\ln$ . For example,  $\ln(x)$  is the natural logarithm of  $x$ .

### Exponential Function

The exponential function is  $e^x$ , and is also written  $\exp(x)$ ; you may see it in either notation on your calculator.

### Relationship Between Natural Logarithms and Exponential Function

The natural logarithm and exponential function are inverse functions, meaning that if you apply one to a quantity, then apply the other one to the result of the first operation, you'll get back the original quantity. Similarly to base 10 logarithms,  $\ln(e^x) = x$  and  $e^{\ln(x)} = x$ .

### Voltage and Current on the RC Circuit

If you first hook up a ohmmeter across the resistor, then apply a constant DC voltage to the circuit, the current will start off at a high value, then decrease exponentially. This is because when the capacitor is discharged it acts as if it has low resistance, so lots of current flows. As the capacitor charges, the amount of current flow decreases gradually (exponentially). At the amount of time called the RC time constant, the value of current that flows through the circuit is only 36.8 percent of the original current. Also, the voltage across the capacitor is 63.2 percent of the applied voltage. The RC time constant is equal to the resistance of the resistor in ohms multiplied by the capacitance of the capacitor in farads. The value of 36.8 comes from approximating the value of  $e^{-1}$ .

The capacitor will never actually reach the same voltage as is presented by the external voltage source. The current flow curve is asymptotic, meaning it gradually approaches, closer and closer, to zero, but never actually gets to that quantity. Similarly the voltage approaches the value of the externally applied voltage, but never quite reaches it.

If the voltage source is then removed from the circuit and is replaced with a conductor of negligible resistance, the decay of the voltage across the capacitor is exponential. In a time period represented by one time constant, the voltage across the capacitor will have decreased by 63.2 percent.

### Practical Assumptions

In practical electronics, if an RC circuit has a constant DC voltage applied for a period of time equal to five RC time constants, the resulting value of voltage across the capacitor is considered to be equivalent to the applied voltage.